The Interplanetary Olympics

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The delegate from Saturn was impressed by the polish on the long table, which shone so brightly in the sunlight that it was almost blinding. Then he remembered that he was on the planet Mercury, and it was the sunlight, not the polish, that was responsible. He looked around as the other members of the Organizing Committee settled themselves into their seats, most experiencing difficulties in the low gravity.

The President of the newly formed Interplanetary Olympic Committee rose to his feet amid a hushed silence. "Ladies, gentlemen: this is a historic occasion. Next year, 2092, will see the 50th Olympic Games of the modern era. And, I need hardly remind you, the very first Interplanetary Olympic Games. This of course poses unprecedented logistic problems. To avoid transporting millions of human beings around the Solar System, it has been necessary to hold the games simultaneously on all ten worlds of the Interplanetary Sporting Union: Mercury, Venus, Earth, Mars, Jupiter, Saturn, Uranus, Neptune, Pluto, and — last but by no means least — the Lunar Enclave."

"For the record, Mr. President," said the delegate from Neptune, "how precisely will that work?"

"The arrangements are simplest for for field events," replied the President. "On each world, competitors will perform under the supervision of local officials. The overall results will be collated centrally. The competitors who put in the best three performances will be awarded medals. In the track events, the same principles will apply, but competitors will run against the clock. The team events are admittedly more problematic, but at our previous meeting we agreed — narrowly — to try artificial-reality simulation links."

The Lunar delegate raised her hand. "Mr. President, my Planetary Committee wishes to raise an objection."

"Typical," muttered the Venusian delegate. "Troublemakers, those Loonies, every one of 'em. It's not even a planet, but its so-called Planetary Committee throws its weight about as if it was Jupiter."

"We consider," the Lunar delegate continued, "that Jupiter's greater diameter will offer its competitors un unfair advantage in the weight-lifting."

"What?" yelled the Jovian delegate. "That's ridiculous!"

"It is well-established that in events requiring sudden effort, competitors do better when the games are held at high altitude," the Lunar delegate explained. "Since altitude is defined by the distance from the planet's centre, the large diameter of Jupiter provides a massive advantage to the Jovians."

"Good heavens, really?" asked the President. "That's amazing."

The Jovian delegate rose angrily to his feet. "Has it occurred to the Lunar Enclave," he said, his voice dripping with irony, "that Jupiter's greater gravity puts its team at a considerable *dis*advantage in the weightlifting? This completely overrides any alleged effects of altitude, which in any case should be measured from sea level, not from the centre of the planet."

"Jupiter hasn't got a sea," the Lunar delegate pointed out.

"Neither has the Moon."

"Yes it has! The Sea of Tranquillity!"

"Ladies and gentlemen," said the President, "may I seize upon that last word. Tranquillity. This meeting must operate in an atmosphere of friendliness and tranquillity, to demonstrate to the entire Solar System the unity of sport and the importance of good sportsmanship."

"And sportswomanship," said the Lunar delegate.

"That's all very well," said the delegate from Saturn. "But you know very well that high gravity makes it a lot harder to throw a javelin a long way on Saturn than on Mars."

"Don't bring Mars into this," objected the Martian delegate. "We've kept a very low profile so far in this debate."

"It's not a debate," said the President, miffed. "It's my opening speech."

"Well, it's a debate *now*," said Luna. "And I move that the results of the weight-lifting competition should be adjusted to compensate for the effect of planetary diameter."

"Seconded!" yelled Pluto and Mercury together.

"But that's idiotic," screamed the Jovian delegate. "Luna has small diameter, *and* low gravity. Anyone can lift *tonnes* on the Moon as it is, and now you want to distort the figures even further in your favour!"

"I must say," Uranus began ponderously, "that I tend to agree with — "

The President sighed. "Fellow delegates," he said, "it is clear that we have run up against a fundamental issue. Local conditions are far from uniform across the Solar System. We really cannot tackle the question piecemeal: the correct compensation for local conditions depends upon the sport. I propose that we set up a small subcommittee to discuss the issue and report back to our next meeting. It should clearly involve Luna and Jupiter, and I propose Venus to represent the medium-sized planets. And assuming

we can agree on that, I will now move on to the much more serious question of the permitted size of sponsors' logos..."

"...so I think we can agree that diameter is *not* an important influence upon weight-lifting," said Venus. Jupiter nodded, and Luna stuck her tongue out at him. "Carried on a majority vote. Moreover, we are agreed that the main influence is *gravity*; and that all masses should be multiplied by the local gravitational force."

"That's what weight is," Jupiter pointed out. "Mass times gravity. If gravity didn't matter, the sport would have been called *mass*-lifting."

"That may be true for the press, but I still think there are problems with the snatch," said Luna. "There the weight has to be lifted rapidly, it's a question of an impulse rather than a sustained force. An impulse is a change in momentum, and momentum depends upon mass, not weight."

"We will note your dissenting view in the final report," said Venus. "Now, track events."

"That's very tricky if you get too detailed," said Jupiter. "It depends upon the athlete's precise gait. The best we can do is analyse a plausible model. I think there's a simplification that will suffice for all events except perhaps the short races. We can consider just the horizontal component of the athlete's velocity. When an athlete runs, chemical energy in the body is converted into kinetic energy of motion; and the important component is that which propels the athlete forward. The chemical energy E that can be generated by a standard model human is the same on all member planets. Moreover, kinetic energy is ïmu^2 where u is velocity and m is mass. Thus E = ïmu^2, so u = œ(2mE), which is independent of the force of gravity. In short, there is no need to adjust for gravity in track events."

"I guess," said Luna. "But there is some up-and-down motion in running too, and that depends on gravity since it involves potential energy as well as kinetic. Anyway, you're neglecting the initial acceleration phase."

"Which is why I think we need to examine the shorter events more carefully. But not right now. As for vertical motion, I think it's up to the athletes to develop techniques to minimise it."

They provisionally agreed to recommend no compensation for the longer track events. "Next, the high jump," said Venus.

"That certainly depends on gravity!"

"Yes, Luna, I agree. I have therefore prepared a table of the values of the force of gravity on the ten planets of the Union." (Table 1).

Table 1 Gravity in the Solar System

Planet	Force of Gravity at Surface (ms^{-2})
Luna	1.62
Mercury	3.70
Venus	8.85
Earth	9.81
Mars	3.72
Jupiter	26.39
Saturn	11.67
Uranus	11.48
Neptune	11.97
Pluto	1.96

"This one's easy," said Jupiter. Think of an an athlete of mass m. He — "

"Or she," said Luna.

"*They* can jump vertically at an initial velocity u, determined by the impulsive force exerted by their muscles, and independent of gravity. Their initial kinetic energy is again ïmu^2. If they rise to height h then they trade it for potential energy mgh, where g is the force of gravity. So mgh = ïmu^2, and h = u^2/2g. That depends on g all right. The bigger g, the lower the jump."

"Can I suggest one small correction?" asked Venus. "What you have calculated is the height to which the athlete's *centre of mass* can rise. But part of the technique of high jumping is to bend the body so that the centre of mass is *below* the bar. I suggest we apply a correction of 0.2m. to allow for that."

"You've forgotten something else," Luna pointed out. "The athlete's centre of mass doesn't start off at ground level."

"No, it's about a metre high."

"Yes, but the knees *bend* before the jump, and — "

"I propose," said Jupiter, "that we apply a correction of precisely one metre. Then the actual height jumped will be h = 1+u^2/2g."

"Can we draw up some comparisons based on that formula?" asked Jupiter.

"Sure. Let's see... on Earth in 1988, just prior to the twenty-fifth Olympics, the Olympic record for the men's high jump was 2.38m, set by Gennadiy Avdeyenko of the Soviet Union." He glanced Luna's way. "And the women's high jump record was

2.03m, set by Louise Ritter of the USA," he added hurriedly. "Using the formula, I calculate that Avdeyenko's vertical velocity was 5.203 ms^{-1}, and Ritter's was 4.495 ms^{-1}. Had they made the same jump on the Moon, they would have reached...hmm, 9.36m and 7.24m respectively. And on Jupiter... only 1.51m and 1.38m." See Table 2 for a complete list.

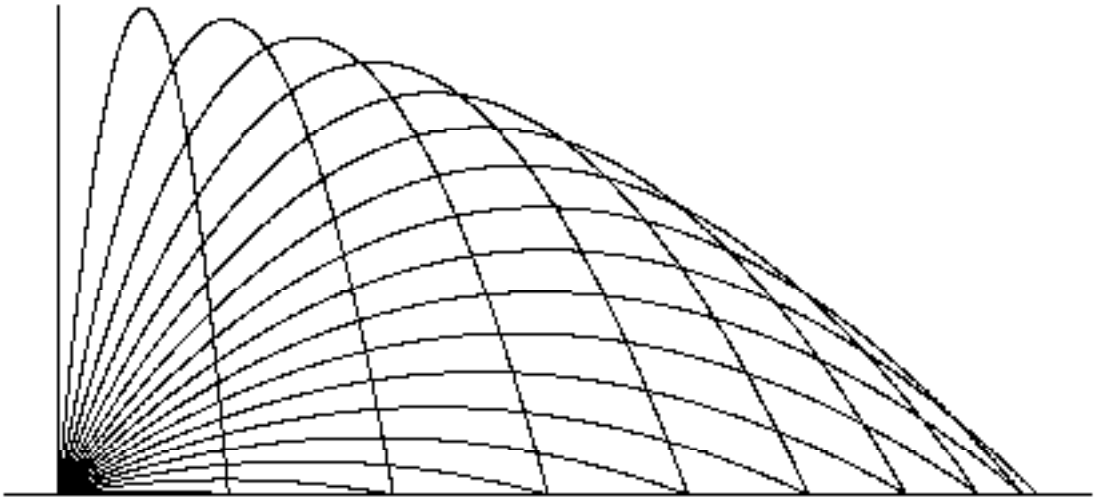
Table 2 High Jumping in the Solar System

Assumptions: Initial vertical velocity is 5.203ms^{-1} for men, 4.495 ms^{-1} for women. Centre of mass rises 1m less than bar height from ground. Forwards motion ignored. No air resistance.

Planet	Height for Men (m)	Height for Women (m)
Luna	9.36	7.24
Mercury	4.66	3.73
Venus	2.53	2.14
Earth	2.38	2.03
Mars	4.61	3.72
Jupiter	1.51	1.38
Saturn	2.16	1.87
Uranus	2.18	1.88
Neptune	2.13	1.84
Pluto	7.91	6.15

"I'd like to tackle the shot put next," said Jupiter.

"Right. Now, it's well known that in order to maximise the distance travelled, the shot should be launched at an angle of 45_," said Venus (**Fig.1**).



A projectile launched from ground level at multiples of 5_. The maximum horizontal distance is travelled when the angle is 45_. This rule no longer applies if the projectile is not launched from ground level or if it has an additional horizontal component of velocity.

"That should help keep the analysis simple."

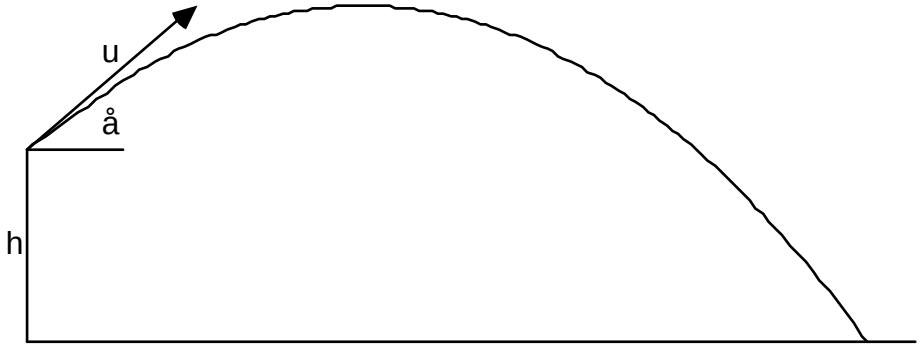
"It would — except it's wrong," said Luna.

"Don't be silly, Luna."

"the 45_ rule is valid only for projectiles launched from ground level."

"Oh."

"And shots are launched from shoulder height, maybe higher." (See **Fig.2**)



A model of the shot put.

"Shouldn't we allow for the diameter of the shot?"

"Well, they sink into the ground when they land, and anyway, our errors are probably bigger than that already. No, we can model it as a point mass."

"Hmm," said Venus. "Let's call that 2m off the ground. Let me see... Suppose the shot is launched at an initial angle å to the horizontal, at velocity u, from height h

above ground. The basic formula for projectiles is s = ut-ïgt^2, where s is distance travelled, t is time, u is initial velocity, and g is gravity. All we do is consider horizontal and vertical motion separately, noting that gravity acts vertically..." There was a lengthy pause while Venus fiddled with the symbolic algebra program on his wrist-computer. "Mmm... I calculate that the maximum distance travelled is $\frac{u}{g}\sqrt{u^2+2gh}$, and to achieve that the angle has to satisfy the eqaution sin å = $\frac{u}{\sqrt{2(u^2+gh)}}$. Ignoring air resistance, of course; and we decided to let h = 2. Now, the men's Olympic record in 1988 was 22.47m, set by Ulf Timmermann of the German Democratic Republic, and the women's record was 22.41m, set by Ilona Slupianek of the same country in 1980. I'll just do the calculations for the men's event, Luna, and leave you to sort out the women's. Assuming the shot was launched at the optimum angle, Timmermann must have launched it with an initial velocity u = 14.2 ms^{-1}. The optimum angle at that speed is 42.46_, a little bit *less* than 45_. A shallower angle that 45_ lets the shot travel further when dropping the final 2m, but that has to be offset against the shorter distance travelled prior to that stage."

"And on Jupiter?" asked that representative.

"Assuming the same value of u, the optimum angle would have been 39.01_, and the distance would have been a mere 9.43m. On Luna, in contrast, th˜ . Ignoring air resistance, of course; and we decided to let h = 2. Now, the men's Olympic record in

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Table 3 Putting the Shot in the Solar System

Assumptions: Initial velocity is 14.2ms^{-1} at the optimum angle. No forward motion of athlete at point of delivery. The shot leaves the hand 2m above ground level. Air resistance ignored.

Planet	**Distance (m)**	**Optimum angle**
Luna	126.45	44.50_
Mercury	56.46	43.99_

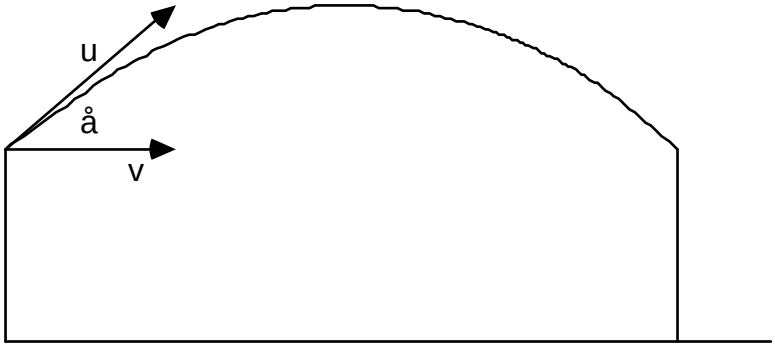
Venus	24.70	42.69_
Earth	22.47	42.46_
Mars	56.17	43.89_
Jupiter	9.43	39.01_
Saturn	19.17	42.02_
Uranus	19.46	42.07_
Neptune	18.74	41.95_
Pluto	104.86	44.45_

"What about corrections for the effect of planetary diameter?" said Luna stubbornly. The others glared at her. "Well," she giggled, "it was worth a try. Old Rattleguts nearly fell for it at the Committee meeting."

"The javelin poses an additional complication," said Jupiter. "The athlete is moving rapidly forwards when the javelin is released, and that imparts extra velocity."

"*And* it's released well above ground level," said Venus.

"I propose we first consider the long jump," said Jupiter. "Though we call it the horizontal jump because on Jupiter it isn't actually very *long*. That has the forward velocity but not the vertical displacement." (**Fig.3**)



A model of the long jump.

"You're forgetting that the athlete's centre of mass is not at ground level, again," objected Luna.

"No, I'm making the approximation that when the athlete's feet hit the sand, the centre of mass returns to its original height."

"But they stick their feet out in front and kind of slide down the parabola," said Luna.

"Crumbs, it's only an *approximation*, Luna!"

"Some of the cycle their feet while in the air," Venus pointed out.

"Can't imagine why," Luna said. "It can't possibly have any effect. They can't walk on air."

"The effect," said Jupiter ponderously, "is largely cycle-illogical." The others sat up, startled. Had Jupiter made a *joke*? Was this the first recorded sample of Jovian humour? Whatever Jovians were, jovial they were not.

"Jupiter's right: we'll start with a simple model and assume the height of the athlete's centre of mass is the same at the ends as it was to begin with. WE can always think about the complications later. Assume that the athlete is moving forward at velocity v at the point of jumping", said Venus. "He — yes, Luna, or she — exerts an impulse that imparts an additional velocity u at an angle å to the horizontal. I then calculate that the optimum angle å satisfies... oh, heck, which button do you push on this thing... $\cos å = \frac{-v+\sqrt{v^2+8u^2}}{4u}$, which curiously is independent of gravity. The maximum distance travelled is $\frac{2u \sin å\ (v + u \cos å)}{g}$, and that does depend on gravity."

The men's long jump record in 1988 was 8.90m, set by Bob Beamon of the USA in the Mexico Games of 1968. The women's record is 7.40m set by Jackie Joyner-Kersee of the USA in 1988. Assuming Beamon approached the take-off point at 10 ms^{-1}, and jumped at the optimum angle, then his initial velocity u works out as 4.076 ms^{-1}. See Table 4 for the men's event: try to work out the women's for yourselves.

Table 4 Long Jumping in the Solar System

Assumptions: The athlete approaches at 10 ms^{-1} and takes off at at the optimum angle at a velocity of 4.076 ms^{-1}. Air resistance ignored.

Planet	Distance (m)	Optimum angle
Luna	53.89	71.17_
Mercury	23.60	71.17_
Venus	9.87	71.17_
Earth	8.90	71.17_
Mars	23.47	71.17_
Jupiter	3.31	71.17_
Saturn	7.48	71.17_
Uranus	7.61	71.17_

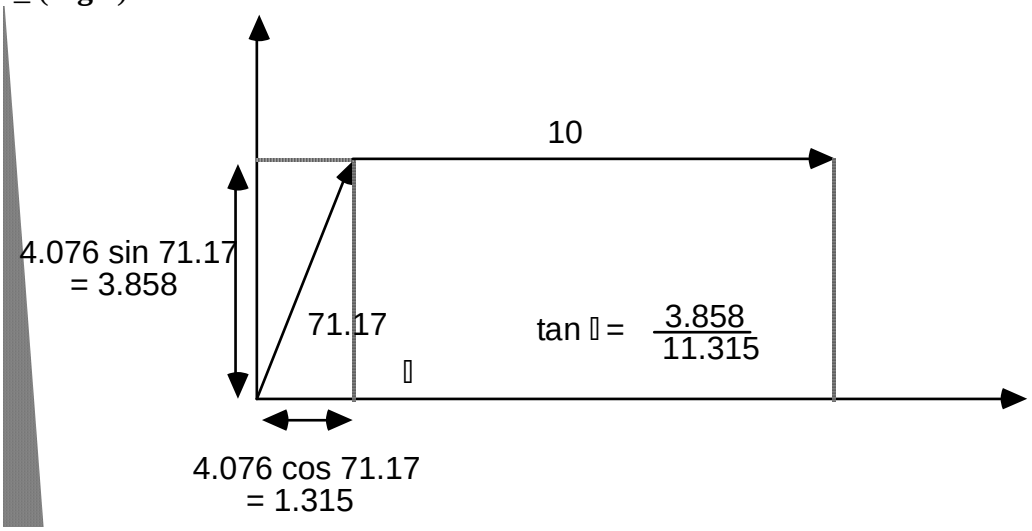
Neptune	7.29	71.17_
Pluto	44.54	71.17_

"That can't be right. They don't *look* like they're jumping at 71 degrees," said Jupiter.

"That's because they're not."

"But you just said — "

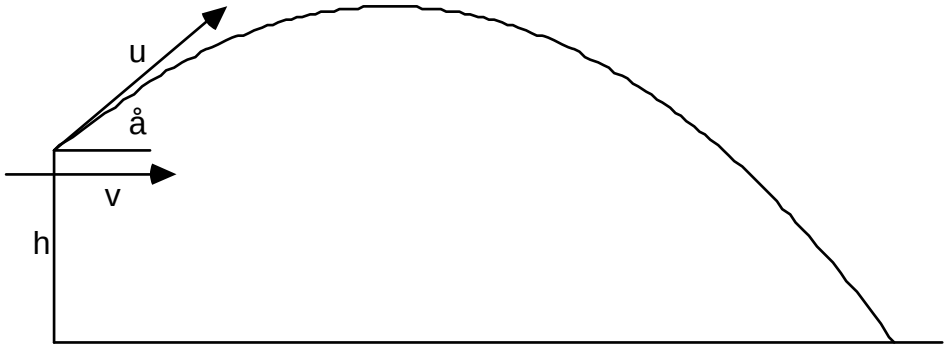
"It's 71 degrees relative to the athlete. But the athlete is moving forwards at $10ms^{-1}$. Beamon's actual launch angle, as veiwed by a spectator, would have been 18.83_ (**Fig.4**).



Computing Beamon's actual angle of attack _ = 18.83_ as seen by a spectator.

Long jumpers follow quite flat trajectories because most of their effort goes into forward motion."

"Now for the javelin," said Venus. "Assume the athlete is running at a forward velocity of v when the javelin is released, that it is released at a velocity of u along a line at angle å to the horizontal, and from height h (**Fig.5**).



Ignore air resistance and the aerodynamics of the javelin. Now the optimum angle... crikey, this is complicated, none of the terms cancel or anything... ah! Change it all into cosines... Mmmm, boils down to a cubic equation. If y = cos å then

$$2uvy^3 + (v^2+2u^2+2gh)y^2 - (u^2+2gh) = 0.$$

You can solve that by a numerical approximation method, and then find å. After which you find that the distance the javelin travels will be

$$\frac{(u+v\cos å)(v + u\cos å)}{g\sin å}$$

."

In 1988 the Olympic record for men was 85.90m set by Jan Zelezny of Czechoslovakia, and the women's was 74.68m set by Petra Felke of the German Democratic Republic. Assuming Felke was moving at 10 ms^{-1} when she released the javelin (admittedly a trifle optimistic), that she released it at the optimum angle, and that it left her hand 2m off the ground, we find that the initial velocity was u = 20.046 ms-1. See Table 5. You can do Zelezny.

Table 5 Throwing the Javelin in the Solar System

Assumptions: The athlete approaches at 10 ms^{-1} and throws the javelin at at the optimum angle at a velocity of 20.046 ms^{-1}, from a height of 2m Air resistance and aerodynamic effects ignored.

Planet	**Distance (m)**	**Optimum angle**
Luna	438.86	53.41_
Mercury	193.66	53.16_
Venus	82.50	52.56_
Earth	74.68	52.45_
Mars	192.63	53.16_
Jupiter	29.34	50.67_
Saturn	63.19	52.24_
Uranus	64.19	52.26_
Neptune	61.67	52.20_
Pluto	363.19	53.37_

Notice that the optimum angle is now *greater* than 45_, unlike the shot put. The reason is that a steeper angle causes the javelin to stay longer in the air, so that extra

horizontal velocity v can act for a greater length of time. However, this has to be offset against the decrease in forward distance travelled due to launching at a steeper angle than 45_. Moreover, as for the long jump, the actual angle at which the javelin initially travels is not å, because of the extra forward component of velocity v. For our model of Felke's throw the actual angle, as seen by a spectator, is 35.58_.

"I'd like to sort out the unisex underwater ski-football next," said Luna...

Early in 2092, the Interplanetary Olympic Committee again met, this time on Pluto. The table still glistened, but with ice, not polish.

"...and it is with great pleasure that I announce that the permitted sizes for sponsors' logos have also been agreed, and you will find them stated in the 700-page handbook included as item 24335.001b/77J* in the agenda," said the President. "I have also taken Chairman's action to accept the joint report of Luna, Venus, and Jupiter on the standardisation of Olympic events." There was heartfelt applause and a great deal of relief all around. "In order to implement that report, I suggest that all records are standardised to the values that they *would* have been, had the event taken place under Earth gravity... Ah, yes, Luna?"

"I must protest the blatant Terrestrialism of that suggestion, Mr. President."

"You have an alternative proposal? After all, the Olympic games did originate on Earth."

"Yes. I propose we tabulate all records in a ten-column spreadsheet, adjusted to all ten local gravitational values. This will facilitate future comparisons for all member planets."

The President's head jerked up in surprise. It was really quite a sensible idea. The motion passed on a show of hands, although the delegate from Earth looked a trifle upset.

"Then, ladies and gentlemen, we may move on to a central item, the catering arrangements for the banquet — *yes*, Luna?"

"Mr. President, there was one event that the subcommittee failed completely to standardise. We referred it back to the full Committee."

"Did you?" asked the President. He had unaccountably missed this recommendation among the ten thousand pages of the report. "Very well, then: consider it we shall. Was it the unisex underwater ski-football, by any chance?"

"Not at all — you'll find that on pages 8,772-9,318. No, it's something much more controversial."

"What?"

"The synchronised swimming," said Luna.