

The Great Drain Robbery

Ian Stewart

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**Mathematics Institute
University of Warwick
Coventry CV4 7AL
UK**

When I entered Holmes's lodging in Baker Street I found him assembling newspaper, firewood and coals. It was snowing a blizzard outside, and the room was like an icebox. I noticed that he had set aside one sheet of paper, torn from a newspaper, and when he finally rose to his feet, clutching the sheet in his hand and studying it like a cat confronted by a mouse, my curiosity could no longer remain within the bounds of polite behaviour. "Holmes," I cried, "what is it?"

"Your powers of observation must be even more feeble than I thought, Watson," said he. "This is a sheet of newspaper. I was laying a fire."

"No, no, Holmes," I protested. "Even I can deduce that. No, I was wondering what vital news had captured your attention."

Holmes essayed a wry smile, and answered me in his typical circumlocutory manner. "I have often observed a peculiar tendency of the criminal mind, Watson."

"What tendency is that, Holmes?"

"More than once in my career have I uncovered the same perpetrator for separate crimes at the identical location. Often many years apart, you must understand. The criminal is initially repelled from the scene of his evil deed, yet it holds such a fatal attraction that he cannot restrain himself and eventually succumbs to its lure. When he returns, some subtle mental chemistry comes into play, and as often as not a second offence takes place."

"You mean, Holmes," I said, trying to take in the amazing novelty of his thoughts, "that the criminal always returns to the scene of the crime?"

"I mean much more than that," he replied. He handed me the torn paper. "Read this, Watson, and tell me what you think."

I scanned the page rapidly. There was an advertisement for throat lozenges, a lengthy article on the forthcoming marriage of the Earl of Sandwold to Henrietta, third daughter of the Duke of Duckingham, and a ludicrous piece about the future of horseless carriages. Eventually my eye caught a single paragraph in one corner, reporting the escape from Dartmoor prison of the murderer Hugh Dunnett.

I recalled the affair well. A certain Miss Beetroot had been murdered at Ghastleigh Grange, ancestral home of the Duke of Ghastleigh. By a chain of impeccable logic, Holmes had deduced that the murderer was the butler.

"I am deeply concerned, Watson, for the Duke's safety."

"Of course! If you are right about the criminal mind —"

"I am *always* right about the criminal mind, Watson."

"I mean, Holmes, that thanks to your deep *understanding* of the criminal mind, the Duke is in deadly danger! We must fly to Ghastleigh Grange at once."

"No, Watson, we must take the train. I have already made arrangements."

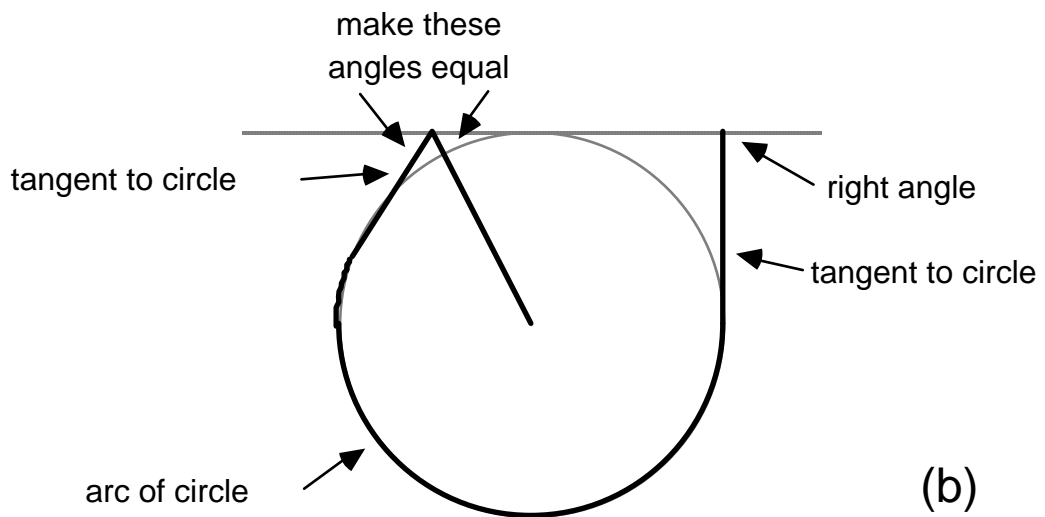
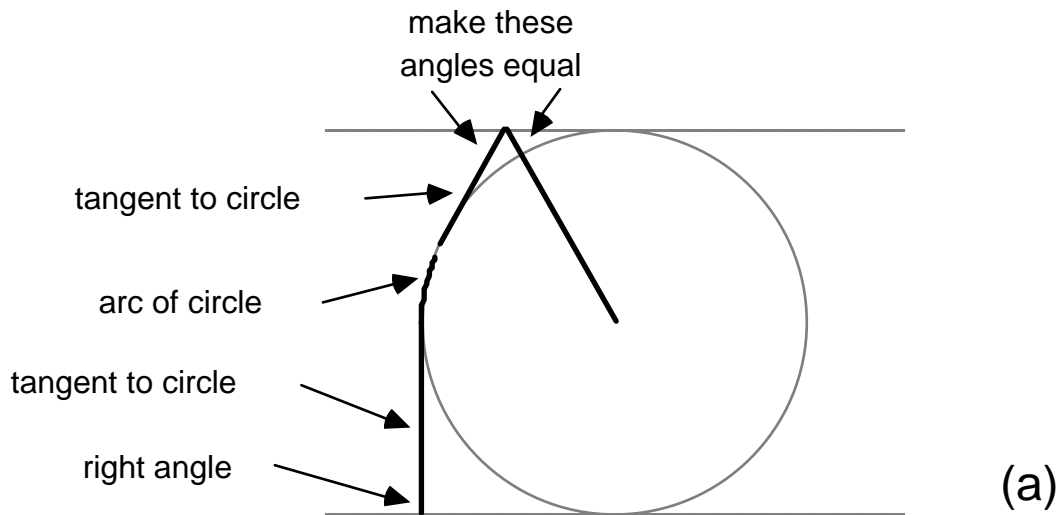
During the lengthy journey Holmes amused himself with his violin, while I endeavoured to read a small volume of mathematical conundrums. "Look, Holmes, here's an interesting one. A man is in the centre of a parallel-sided river 200 yards wide when suddenly a fog descends and he loses all sense of direction. What is the shortest path he can take that will minimise the longest time taken to reach land?"

"He can deduce the direction to the bank by observing the flow of the river," said Holmes, "and then swim at right angles to that in a straight line."

"No, he can't — I mean, suppose the river isn't flowing, or it's a lake or something."

"If you insist. What is the path, then?"

"Nobody knows. But they *think* it's one that goes straight for a little over 100 yards, then makes a sharp turn left, followed by a straight bit, a curved arc, and another straight bit (**Fig.1a**). There's a similar problem when the swimmer is at sea and is 100 yards from a straight shoreline (**Fig.1b**), and again the answer is only a conjecture."



Conjectured best paths to be followed by a swimmer lost in fog. (a) When the swimmer is in the middle of a river with parallel straight sides. (b) When the swimmer is a known distance from a straight seashore. (Each dotted line shows one possible position of a bank or the shore: the actual position is some rotation of the line shown.)

"Fascinating," said Holmes, with sarcasm barely concealed. "I am glad you are interesting yourself in practical matters, Watson." He returned to his violin and I tried to continue reading, but my pleasure had been blunted by his disapprobation.

Upon our arrival at the Grange a maid conveyed us to the Duke's suite. He looked pale and haggard, as if he had suffered a sleepless night. As indeed he had. "Thank you,

Lucinda, you may leave us now. Holmes, I am delighted to see you," he said. "The Ghastleigh Goat went missing last night. I fear it may have been stolen."

Holmes endeavoured to calm him, and eventually began to piece the story together. The animal in question was a family heirloom, brought from India by the tenth Duke at the end of his successful campaign against the Mad Maharajah of Marzipur. It was made of bronze and worth virtually nothing. But the animal had a hollow belly with a secret drawer, inside which a number of important documents were traditionally kept. When Holmes enquired the nature of those documents, the Duke's face went even paler. "I cannot reveal their contents, Holmes. It is an ancient stain on the family escutcheon. If the matter were to be made public, it would be the end of the Ghastleighs."

"Was Dunnett aware of the goat's secret?"

At this, the Duke's face reddened. "That fiend who ingratiated himself for twenty years and was finally revealed as a foul murderer? My erstwhile butler? He certainly could have known that the goat contained documents of a delicate nature. He had no knowledge of the method for opening its concealed drawer, though — it is a family secret, passed on from one Duke to the next. However, he would be able to hack the animal apart — though with considerable difficulty."

"Then you are indeed in trouble, for Dunnett has escaped." The Duke froze in horror. "Holmes, I am ruined."

"Not yet," said the great detective. "Show me the room where the Ghastleigh Goat was kept."

The Duke called Lucinda the maid and told her to fetch a lantern, and we set off through the Grange's maze of corridors.

It was a small, draughty cellar, decked with cobwebs, its sole light coming from a rusty iron grille in the ceiling which led to a narrow opening at ground level. There was an unpleasant smell, and the floor was inches deep in dust. Even I could see innumerable footprints. In one corner was a large safe. "The goat was in there," said the Duke.

Holmes studied the floor, his eyes following the trails of footprints. He took out a magnifying glass and walked across to look closely at the grille. He inspected the lock on the cellar door and the safe in a similar manner. Falling to his knees, he searched the dust until he found a small paper wrapper, which seemed to adhere to his fingers. He sniffed the air, glanced towards a pile of old cardboard boxes. "How large was the goat?"

"Quite big," said the Duke, holding his hands about three feet apart.

"Then the whole story is here, your grace, for anyone to read who comprehends the runes of observation. At first I feared the goat might have flown the nest, but now I believe there to be a small element of hope."

The Duke's eyes lit up. I gave Holmes a meaningful look, and he explained his reasoning. "The thief — I am sure it was Dunnett — made his entrance and exit through the grille. The cellar door is untouched, I am sure of it. He unlocked the safe and removed the goat. Lacking knowledge of how to open its secret drawer, and taking into account the difficulty of cutting it open in this cellar, as well as the danger of discovery, he then removed it."

"But how did he get it out?" asked the Duke. "It must have been a tight squeeze for the man to get in, and the goat is considerably larger. It would never get through the grille."

"Ah. He attached it to an inflatable rubber tube, for buoyancy, and dropped it into the drains, to float away and be collected outside the Grange's grounds."

"Holmes, that's ridiculous," I told him. "You can't possibly know that. Anyway, there is no drainhole in this cellar."

"Watson, you underestimate my powers, as usual. Upon the floor I found the remains of a puncture repair kit of the kind used for bicycles. Obviously the tube suffered a puncture while being pulled through the grille, and repairs had to be made on the spot. The smell that you cannot have failed to notice indicates that there is a drain nearby. As for the lack of a drainhole — see for yourself." Holmes kicked away the cardboard boxes, and a large flagstone appeared with two iron rings in it. "It had to be there, from the way the footprints led. But I have reason to believe the thief was unlucky, Watson. I have made a lifelong study of the smells of drains — you perhaps recall I published a small monograph upon the subject — and this one smells as if it has recently been blocked. Very recently, probably last night. Now, Watson, if you will lend me your considerable physical strength, I think we can lift this flagstone. Good. Pass that lantern, will you? Yes, as I thought. Look."

I saw a deep shaft, lined with stone, about a yard square. At the bottom, a good forty feet below us, noisome slime lay stagnant.

"The shaft is surprisingly deep, seeing as we are in a cellar."

"The ground rises near the Grange. This cellar is above much of the surrounding land."

I peered into the shaft. "I see no sign of the goat."

"No. The drains were flowing when the animal was dropped in. But somewhere on its journey out, the improvised puncture repair gave way and the tube deflated. The goat then sank to the bottom of the drain, partially blocking it. Other material became caught, and this completed the blockage."

"So the documents are trapped somewhere in the drains?"

"Precisely. But the shaft is too deep and dangerous for anyone to attempt to locate the blockage from this end. We must break into the drainage system at a more convenient point. Do you have maps?"

"In the library," replied the Duke. "Let us proceed there at once."

Unfortunately, it was not that easy. None of the maps showed a drain that might plausibly connect to the cellar.

"Drat," said I. "The scoundrel will even now be making his way back up the drains from the exit, seeking his booty."

"I think he will wait for the cover of night," said Holmes. "That has always been his *modus operandi*. So we have a little time yet." He paused, deep in thought. "When we arrived, I saw an elderly gentleman hoeing in the carrot-beds."

"Old Ned, you mean. Deaf as a post, but a good servant. Been with us for ages."

"Perhaps he can recall the layout of the drains. Gardeners often remember that sort of thing. Then we could locate the exit, conceal ourselves, and wait for Dunnett to arrive."

After a lot of gesticulating and shouting, Holmes managed to explain to Old Ned what was wanted. "Ar, yur," said Ned. "There wur a big old drain ran roight across the front lawn in a dead straight line. Then it wiggled about a bit and disappeared. Rumour was it comed up in the cellars. You could tell it wur there because the grass always changed colour in Summer."

"So at that point it was near the surface," Holmes deduced. "That makes sense. As I remarked, the Grange is set on a slope and the cellars are higher than the lawn."

"The front lawns are *huge*," said the Duke.

"Can you show us where the drain runs?" asked Holmes

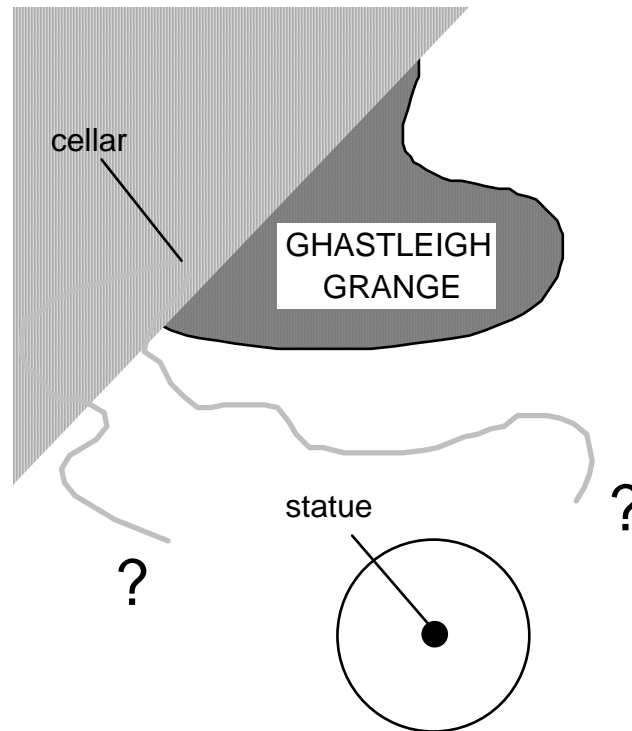
"You mean, loike, today? Ar, no, it be winter now, and covered in snow. But I do recall it passed within a hundred yards of the statue of the water-nymph."

"We must dig a trench," said the Duke. "I will summon every available man."

"We must also dig it *fast*," said Holmes.

"And the right shape," said I. "For instance, if we dig a straight trench, it might by misfortune run parallel to the drain. Or roughly so, in which case it might not meet the drain in the time available."

"What we need to know," said Holmes, "is the shortest trench that is guaranteed to meet *every* straight line that passes within a hundred yards of the statue of the water-nymph (**Fig.2**)."



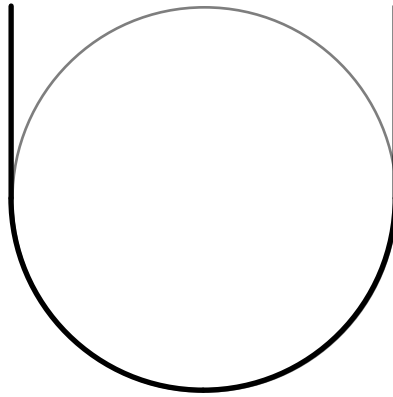
Part of the grounds of Ghasstleigh Grange. The drain, which is straight, passes through the marked circle. But where?

"We could dig a circular trench with a hundred-yard radius," the Duke suggested.

"With a length of 200_ yards, or about 603 yards," Holmes rapidly calculated.

"I doubt we have time for such a long trench," said the Duke. "My men could get close to that figure, though. Can we do better?"

I had a sudden recollection, as if struck by a flash of blinding light. "Holmes! The book, the book I was reading on the train." I pulled it from my coat pocket. "Let me see... yes, here (**Fig.3**).



The shortest ditch that will always intersect the drain consists of a semicircle and two tangential straight lines of length equal to the radius.

The shortest path that meets every chord of a circle of radius r consists of two parallel straight segments and a semicircle —"

"With total length $(2+\pi)r$, or about 514 yards in this case," said Holmes.

"A saving of 89 yards," I said.

"I believe we can manage that," said the Duke. "There is no time to lose: I will set the men to work."

While they worked, Holmes took a look at my book. "I confess I misjudged the utility of this slim volume, Watson," he said. "But, out of interest, how do we *know* that the path proposed is truly the shortest?"

"Why, it has been proved beyond any shadow of doubt by mathematical means," I told him. "By several different people, under slightly differing conditions on the nature of the curve allowed, but first by H.Joris."

"Can I see this proof?"

"All known proofs are surprisingly long and complex," I told him. "It would be of great interest if anyone could find a short, simple proof." We took to discussing other variations on such problems — a man lost in a rectangular forest of known dimensions, a skater on a frozen elliptical lake when a blizzard comes down... We watched the diggers, their spades flashing in the sunshine as the shadows lengthened. Suddenly Old Ned let out a yell. He had located the drain. A few minutes' work made its direction clear. Holmes sighted along it. "Ah, yes, beyond the coppice in the distance, where the stream flows. That is where we shall find the outlet. And the thief."

We secreted ourselves among the trees, and settled down to wait. No sooner had the sun set than we heard footsteps, and the sounds of splashing. A masked figure came into view, and Holmes leaped on him. There was a brief struggle, and then the criminal

was lying on his back and Holmes was sitting on top. Holmes seized the mask. "Now we shall see the criminal," he cried. "As I deduced at the start, it is — who the devil are you?"

"This isn't Hugh Dunnett?" I said in astonishment.

"It *is* who dunnit," said Holmes with some heat.

"My god, it's Lucinda," said the Duke, who had just arrived on the scene. "What are you doing here, my dear? Don't tell me *you* are our thief?"

"God please your grace, no. Yesterday evening I was sent to the cellar to sort out some boxes of junk, but the door was locked and I couldn't find the key, so I climbed in through the grille. There was an open drainhole with a stone slab beside it. The safe was open and I saw this funny old goat statue thing in it, and I pulled it out to take a look. It was heavier than I expected, and it accidentally fell down the drain. A lot of junk went in with it, and the drain got blocked. I looked down and I couldn't see the goat, so I assumed it had floated away before the blockage occurred."

"Lucinda, it's made of *bronze*."

"I thought it was wood with gold paint, sir. Anyway, I panicked. I pulled the stone over the drainhole, shut the safe, and climbed out again. I found a map in the library showing where the drains went — it's hidden in my room, I swear I always intended to give it back once I'd — anyway, I was about to crawl up the drain looking for the goat when this gennelmun —" she gave Holmes a winsome smile — "leaped on me."

"So the Ghastleigh Goat is sitting at the bottom of the shaft," the Duke mused, "and I have a five-hundred yard trench in my lawn. Dunnett remains at large, but appears not to have been involved in any way." He gave Holmes a hard stare.

"It is a question of logical deduction," said Holmes. "As I have often stated, when you have eliminated the impossible, then whatever remains, however improbable —"

"Yes, Holmes, yes! Go on!" I cried. "That's brilliant!"

"— remains improbable," said Holmes, deflated. "There's probably something altogether different going on, and you've missed it. But don't quote me on that," he warned.

"My lips are sealed," I replied.

But my pen and notebook are not. After all, a biographer has to make a living.

FURTHER READING

Hallard T Croft, Kenneth J Falconer, and Richard K. Guy, *Unsolved Problems in Geometry*, Springer-Verlag, New York 1991.

Ian Stewart, Murder at Ghastleigh Grange, *Scientific American*, October 1992, 95-97.