

Turning the Tables

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Sixty-seven floors up in the Ruff Tower, two employees of We-Haulit-4U Moving struggled with the last of nine solid oak tables, every one of which had been carried by hand up the narrow twisting staircase that was normally reserved as an emergency fire exit. They would have used the elevator but Goofy Ruff, the owner of Ruff Towers, was worried that the tables' weight might be too much for the suspension cables.

They dragged the table into the storeroom to join the other eight. The door clicked shut behind it.

"Done," said Dan, breathing heavily. "Final check, and then I'll buy us lunch at the Plushy Pink Pizza Palace. Two jumbo-sized square tables, six humongous rectangular tables, and one top-of-the-range megabronto."

"Right," said Max, ticking off the items on a scruffy clipboard. "One one by one, six two by ones, and one two by two." His pen scribbled across the page. He looked up. "Say, it's a bit crowded in here."

"Jam packed. Wall-to-wall tables, except where we're standing."

"We did well to fit them in. I wonder why they want so many?"

"I think they're just using this room for temporary storage until the redecoration of the ballroom on the ground floor is finished. Word is that Rasputina Ruff told Goofy she really preferred lime green to turquoise—"

Max groaned. "You mean we carried this stuff all the way up here and they're going to want it back down there?"

"Yup. Next week. It's business, Max, don't knock it. Think of it as a challenge, a test of mental character and physical strength. I can't resist a challenge, can you?"

"Oooh, I have enough mental character and physical strength to resist *any* challenge. You know, I think I'll get a job digging drains, it's nearer the ground."

"Speaking of which, so is the Plushy Pink Pizza Palace."

"Right. Let's go. Woops."

"Whaddya mean, woops?"

"The door must have locked itself behind us."

Dan's face was a picture. He gathered his wits. "No need to panic, there's supposed to be an emergency phone somewhere."

"I know," said Max. "It's behind that small door in the wall marked 'emergency phone'."

"Great."

"Which is blocked by that solid oak table."

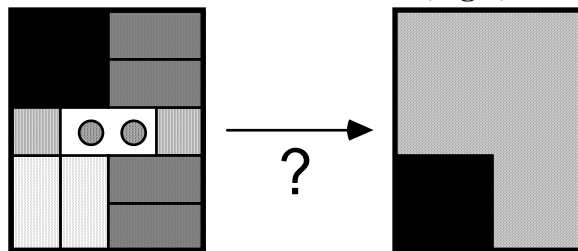
"Not so great. We'll have to move it."

"They're packed pretty tight," Max observed. "It's not going to be easy."

"Can't we pile 'em up somehow, make a bit of space?"

"Not a chance. Ceiling's too low."

After half an hour of futile effort, they called a halt. "Dan, we've got to think this one through before we run out of energy. I reckon we'd be OK provided we could move the megabronto table into that corner over there (**Fig.1**).



Can you slide the tables until the big black square moves to the lower left corner? Dan and Max, marked by two circles, are standing in the only open space.

We can slide the tables into the space that's left, one by one, thereby creating new spaces to slide more of them into."

"Won't we get trapped?"

"No, we can crawl underneath," said Max.

Dan stooped to peer under a table. "You're right, there's plenty of room." He scratched his head, thinking. "You know," he said, "when I was a kid I used to have a toy. It was called 'Dad's puzzler' and you had to slide rectangular and square blocks around so that Dad could move his piano. It was quite like this." He paused. "Suspiciously like this, in fact. Anyway, it took me a while, but in the end I learned how to solve it."

"Great! Can you remember how?"

"Yeah. You slide the blocks around until you get them where you want."

Max grimaced. "I think we need something a little bit more specific, Dan."

Dan shrugged, it wasn't his fault that he couldn't remember how to solve a puzzle he'd been given for his sixth birthday. "I can still recite the whole of *The Cat in the Hat*," he said, by way of demonstrating that he was nevertheless possessor of a super-power memory.

"Yeah. And all we could do was to sit sit sit sit. And we did not like it, not one little bit. Thanks a bunch, Dan."

"No use moping. Let's shift a few of them and see where it gets us."

Another half hour passed, after which they had successfully moved the megabronto table from the top left corner to the middle of the right-hand wall (**Fig.2**). It was progress, but — as Dan remarked — in which direction?

"What we need," mused Max, " is a map."

"Max, we can *see* where all the tables are."

"Not a map of the *room*."

"What, then?"

"A map of the *puzzle*."

Dan stared at him. Have you gone crazy? Puzzles don't have maps."

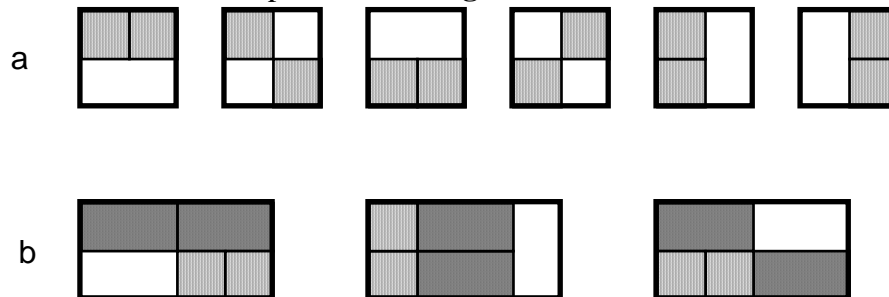
"I hate to contradict you, old buddy — no, come to think of it I like to contradict you, but anyway — puzzles do have maps. Conceptual maps. Imaginary maps in the brain. Maps that tell you what all the positions in the puzzle are, and how to get from one to the other. Mental mazes that tell you what moves to make, and in what order."

Dan nodded. Of course. Except... "It's going to be a pretty complicated map, Max. There's an awful lot of positions, and an awful lot of moves."

"True. So we'd better find some way to cut them down. Break the problem into simpler pieces. Hey! Yes, that's it. First of all, let's find out what we can do *easily*. Then we can try to kind of string those together."

"Well, for a start, if you've got a square hole with just the two smallest tables in it, you can move those tables around pretty freely," said Dan. See **Fig.3a**.

"Yeah, that's the kind of idea. A sort of 'sub-puzzle' where you move only a few tables around, inside some well-defined boundary." (Positions 5-6-7 of Fig.2 use just such a sub-puzzle.) He stopped and thought. "Hmm. There's another, a bit more complicated, where you have a rectangular region containing two rectangular tables, two square ones, and the rest free space." See **Fig.3b**.



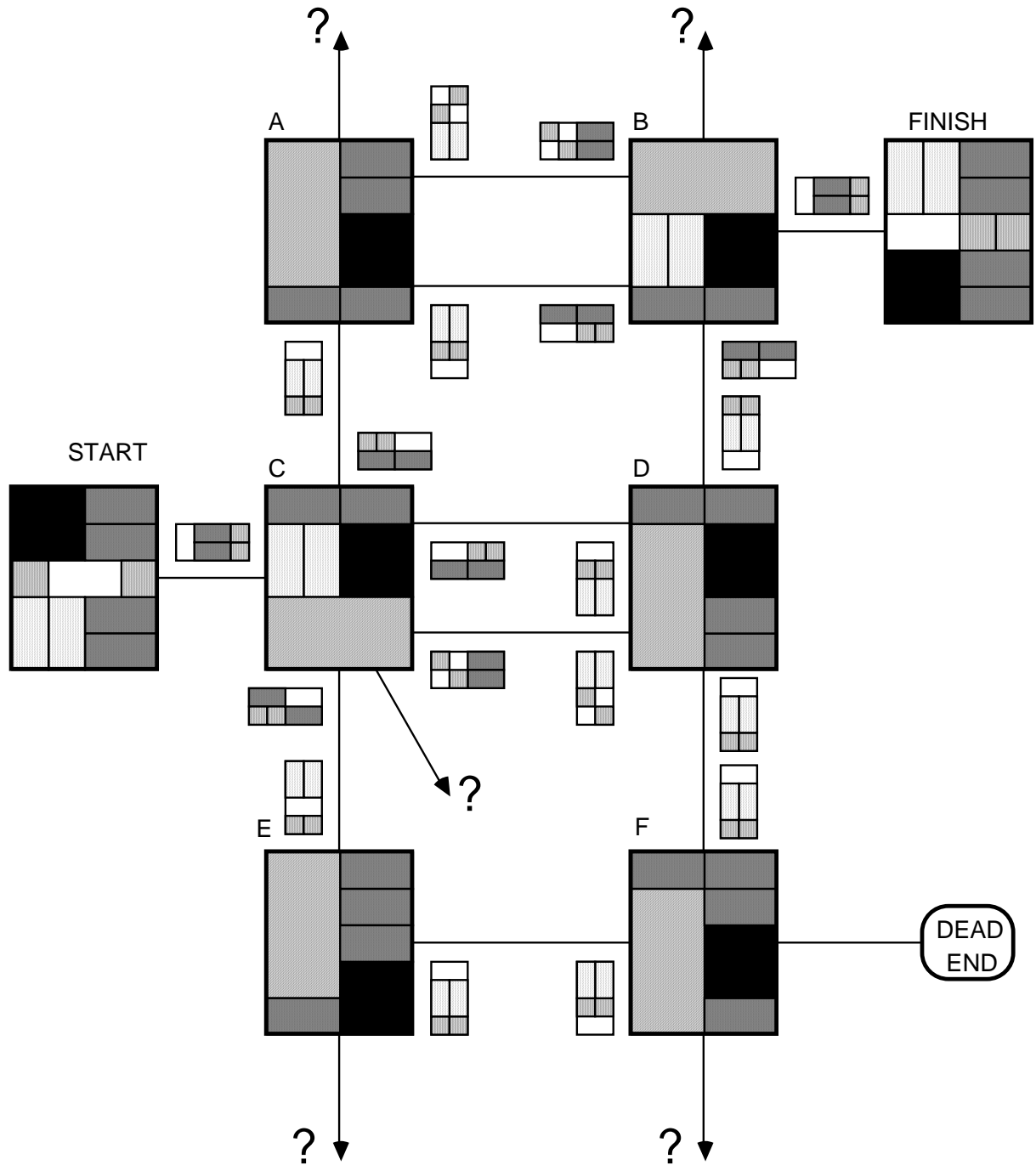
Some useful maneuvers. In each sub-puzzle the tables can easily be rearranged without going outside the boundaries marked.

"So you could assume that positions that differ from each other by shuffling tables around inside one of these sub-puzzles are effectively the same," said Dan. "That must cut the list of positions down quite a bit."

"Yeah. And there's another thing. Sometimes there's only one sensible way to continue moving the tables, if you don't want to just go back on yourself." (Positions 3-4-5 in Fig.2, or the longer sequence 7 through 17, are examples.)

"So provided you know where you're starting from and where you're trying to go, sequences like that can be left off the map?"

"Precisely. Hand me that clipboard and pen." Shortly, Dan and Max were staring at a map of part of the mental maze of possible positions and moves (**Fig.4**).



A partial map of the puzzle. Large diagrams show the placing of key tables. Cross-hatched regions represent sub-puzzles to be solved using the useful maneuvers of Fig.3. Lines indicate sequences of moves, sometimes quite long, that are essentially 'forced' if you know where you want to go. For example the line from START to C represents the sequence of 17 moves shown in Fig.2. The small diagrams show how the sub-puzzles are to be arranged at the start and finish of these sequences. Using this map as a guide, the puzzle becomes relatively easy to solve.

"I've marked the start and the finish positions," said Max. "Then there are various ways to place key tables, which I've marked A, B, C, D, E, F."

"I'd have expected there to be more than just six of those."

"There are. This is just *part* of the map. But it's more than enough to solve the puzzle. Now, shut up and listen. The lines show sequences of forced moves — in the sense that if you know where to start and where to finish, the moves in between are fairly obvious because on the whole there's only one choice you can make at each step, right?"

"OK, I see that. Once you've played around with a puzzle for a while, you can't help noticing that kind of thing."

"You said it. Now, I've shaded in rectangular regions where there's a sub-puzzle to solve. To show *which* sub-puzzle, I've drawn little pictures of the start and finish positions within the rectangle, at the appropriate ends of the connecting lines."

Dan's mouth opened like a goldfish's. "Sorry, I don't quite follow."

"Well, suppose you want to work out how to move from C to E. Look at the vertical line that joins them. Beside it are two little diagrams. If you replace the shaded area in C with the top diagram, and the shaded area in E with the bottom one, that gives you the start and finish positions. Because the moves in between are 'forced', it doesn't take very long to work them out. If you make a copy of the puzzle out of bits of card, you can move them around and check."

"What does DEAD END mean?"

"What do you *think* it means? Now, what does the map tell us?"

"Where things are and how to go between them. Well, clues to those things."

"It tells us more than that. It tells us that one way to solve the puzzle is to go along the route START-C-A-B-FINISH. Just use the little diagrams beside the appropriate lines to fill in the shaded bits of the big diagrams, then follow the forced sequences of moves."

Dan's face lit up in admiration. "You could go START-C-D-B-FINISH instead?"

"Sure. Or even START-C-E-F-D-B-FINISH — but that would be an unnecessarily complicated route."

Dan was getting into it now. "Or START-C-D-F-E-C-D-B-A-B-D-C-E—"

Max interrupted before his friend collapsed from lack of breath. "Yeah. But that would be an even more unnecessarily complicated route."

"I'll settle for the simplest one."

"Fine by me. Let's get these tables moving!"

It took a while to get into the swing of it, but once they did, it didn't take long to get the megabronto table into the lower left corner of the room. After that, Max was able to get his hands on the emergency phone and call the porter in the lobby. When help arrived it turned out that the new arrangement of tables was blocking the door so it wouldn't open, but by now Dan and Max knew their way around the map of Dad's puzzler blindfolded.

Not long after midnight, they were free.

Somewhat shaken by the experience, they flagged down a cab and set off for the Plushy Pink Pizza Palace, which was an all-nighter. First they'd catch up on lunch, then they'd have dinner.

"You know," said Dan, "that wasn't so hard."

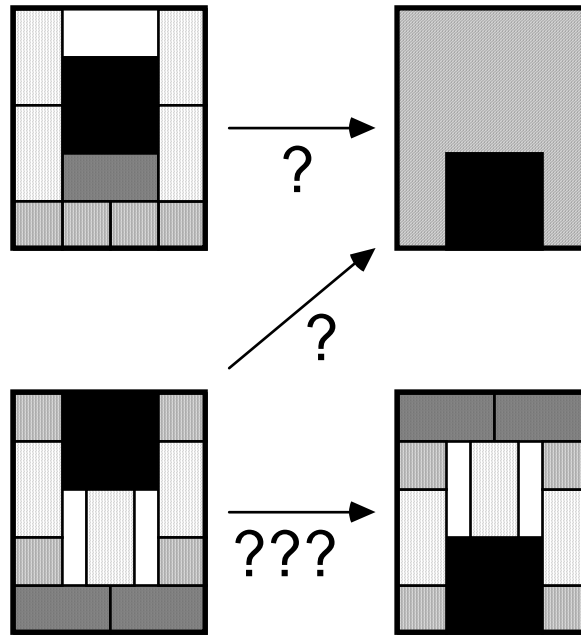
"Not once we'd worked out that map. But we were lucky, it was quite a simple one."

"Yeah. But that's because you used some tricks to simplify it."

Max rubbed his chin — to find a heavy growth of stubble. "The tricks help, but there are plenty of sliding block puzzles with much more complicated maps, even when you use every trick you can think of."

"Like what?"

"Well, there's one called the Donkey puzzle, which is probably from the nineteenth century and almost certainly French. That's a good bit harder. The Century puzzle, invented around 1980, is harder still. You must make 100 moves to solve it. And if you insist that the finishing position should be like the start but upside down, then it's *really* hard. That version is called the Century and a Half puzzle because it requires 151 moves." (See **Fig.5.**)



Three harder sliding block puzzles. Any arrangement of blocks is allowed in the cross-hatched regions. Top arrow: the Donkey Puzzle. Middle Arrow: the Century puzzle. Bottom arrow: the Century and a Half puzzle.

The cab screeched to a halt outside the Pizza Palace; Dan paid the driver. They went in and sat down. Max ordered a deep pan pizza with extra cheese. Dan ordered a special with a lot of extra toppings — pepperoni, tuna, capers, anchovies, beef, pineapple, hot tamales, a whole banana, chewing-gum, liquorice, and a lighted sparkler. "My favourite," he explained to a bemused waitress. "Make sure you build it from the bottom up in the order I specified."

The pizzas arrived. Dan's didn't look quite right. Most of the ingredients were upside down, including the crust. The waitress had included a whole tuna and tried to set the liquorice on fire.

"Enjoy your puzzle, sir," she said over her shoulder.

"Send it back," suggested Max.

"No, no, you heard what she said. I can't resist a challenge, it's a test of character. The pizza just needs rearranging." Dan picked up the tuna, tried to find somewhere to put it while he blew out the liquorice. Where had the pepperoni gone? Oh, yeah, inside the pineapple. The plate just wasn't big enough... He sighed, put the tuna back, and was on the point of calling the manager to complain that his pizza was too difficult to solve.

Then straightened his back, squared his shoulders, and reached for the clipboard.

"What are you doing?" asked Max.

"I can fix it. Just wait till I've made a map of this pizza."

FURTHER READING

Elwyn R. Berlekamp, John H. Conway, and Richard K. Guy, *Winning Ways* vol. 2, Academic Press, New York 1982.